

## Abstract

The aim of this paper is to study the properties of the polyharmonic functions on the union of the rotated unit Euclidean balls. In the paper we examine the Dirichlet type problem for the polyharmonic functions, where the boundary conditions are given only in the terms of the solution. As an application we get among others the mean value property for the polyharmonic functions. Next we introduce and develop the notion of spherical polyharmonics, which are a natural generalisation of the spherical harmonics. In particular we study the theory of zonal polyharmonics, which allows us, analogously to zonal harmonics, to construct Poisson kernels for polyharmonic functions on the union of rotated balls. We find also the representation of Poisson kernels and zonal polyharmonics in terms of the Gegenbauer polynomials. Furthermore, we consider the polyharmonic Bergman space for the union of the rotated unit Euclidean balls. Using so called zonal polyharmonics we derive the formulas for the kernel of this space. Moreover, we study the weighted polyharmonic Bergman space. By the same argument we get the Bergman kernel for this space. Finally, we show the connection between the Poisson kernel for polyharmonic functions on the union of rotated balls and the Cauchy-Hua kernel for holomorphic functions on the Lie ball.