ABSTRACT

The main results of this dissertation are related to the characterization of the Stokes phenomenon for homogeneous linear partial differential equations and their generalizations into moment-differential equations, whose formal solutions are summable in all, but finitely many singular directions. We conduct our research in three different ways using:

1. residues – we consider the Stokes phenomenon for the complex heat equation, when meromorphic initial data $\varphi(z)$ has a simple pole at the point $z_0 \in \mathbb{C} \setminus \{0\}$. Next, we extend the obtained results to initial data $\varphi(z)$ given by a meromorphic function with finitely many poles. Then, we transfer the proven results to the generalization of the heat equation.

2. hyperfunctions – first we define jumps across the Stokes lines in terms of hyperfunctions, then we return to the heat equation and using hyperfunctions we determine the Stokes lines and describe jumps across these lines in the case when $z_0 \in \mathbb{C} \setminus \{0\}$ is not a pole this time, but a single-valued singularity or branch point for initial data $\varphi(z)$. We also give some interesting examples, related to the theorems we have proven. Next, we study the special cases of moment partial differential equations (two complex variables) with constant coefficients, whose initial data are holomorphic on $\mathbb{C}$ but a finitely many singular or branching points. Then, we examine the partial differential equation with variable coefficients depending on the time variable t – we give an integral representation of the multisum of the formal solution, we conclude what Stokes lines are and we derive the formula of the jumps.

3. resurgent functions – for the last time we return to the heat equation and its generalizations to find the Stokes lines and to describe the jumps in terms of resurgent functions. We also present a few examples i.a. when the initial data has a simple pole or an infinite set of singular points.